Ratio

Question:

What is ratio and how is it different from rate?

Possible answers to the question are:

1. Ratio is about comparing this to that. It is the same as rate.
2. Ratio is stuff like 2 : 3 or 2 to 3.
3. Ratio is when you divide the top by the bottom of a fraction.
4. Ratio must be the same as a rational number because 'ratio' is the first part of the word 'rational'.

Response 1 is almost appropriate. A ratio is a comparison of things. For example, the ratio of boys to girls in a classroom might be 2 to 3, which means for every 2 boys there are 3 girls. A rate is also a ratio but it is not quite the same thing. A rate is a special ratio where different **types** of units are compared. Here are some examples:

- **Speed**: Lisa ran at the rate of 10 km per hour (units types are distance and time).
- **Cost**: 12 cobs of corn for $2.50 (unit types are objects and money).

When the second term of a rate is 1, then it is called a unit rate (e.g. typing 40 words per minute: 40 to 1; e.g. making $10 per hour: 10 to 1).

Response 2 concerns a notation system for ratio. There are three common ways to "say" ratio. Colon notation is used (e.g. 2 : 3). Fraction notation is used (e.g. 2/3). Language is used (e.g. 2 to 3 or 2 for every 3).

Response 3 is a procedural response that is appropriate only when a ratio is represented by fraction notation AND when you want to change the ratio to a unit ratio. For example, consider the ratio 4/5 (4 to 5). To change this into a unit ratio, you would divide 4 by 5, getting .8. The unit ratio would then be: .8 : 1 (.8 to 1).

Response 4 is appropriate in the sense that one notation system for ratio is fraction. However, the core meaning of fraction involves a part-whole relationship (e.g. eating 1/2 of a pizza means eating 1 out of two equal parts of the whole pizza). The core meaning of fraction cannot describe a part-part relationship but ratio can.

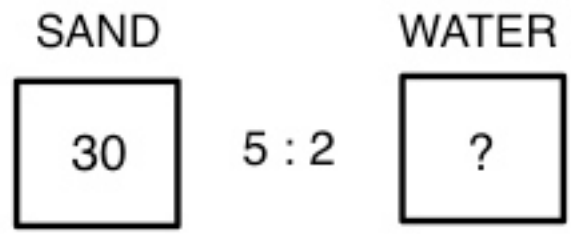
What is a part-part relationship? Consider the situation in the diagram. There are 5 circles and 2 squares. There are two part-whole relationships present: 2 to 7 (squares to all) and 5 to 7 (circles to all). There is one part-part relationship: 2 to 5 (squares to circles). [Note that 5 to 2 is simply a flip of 2 to 5 and is not a different part-part relationship.]

A useful teaching model for developing ratio understandings is the container model. It is comparable to a number sentence for an arithmetic operation. The container model provides a visual sense of the problem at hand.

Consider the following situation.

> George is making a diorama. He wants to include quicksand in it. He researches the ratio of sand to water for making quicksand and finds that it is 5 cups sand for every 2 cups water. He has 30 cups of sand available and wants to use it all. How many cups of water will George need to add to the sand?

The container model that illustrates the problem is shown here. It provides all the necessary information. The '?' indicates what needs to be figured out.



How can the answer be found?

One way is by making a table of values and dumping in the cup amounts (always 5 sand for every 2 water) that need to be added at each stage. This approach is a good way to begin with students because it mimics what someone might actually do to make such a mixture.

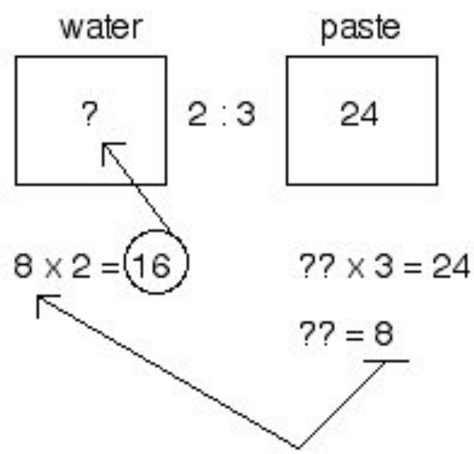
SAND	WATER
5	2
10	4
15	6
20	8
25	10
30	BINGO: 12 cups

One issue with the table approach is that it suggests that ratio concerns additive thinking, when in fact it does not. Ratio concerns multiplicative thinking. This is one reason why a short cut method that involves multiplication should be developed.

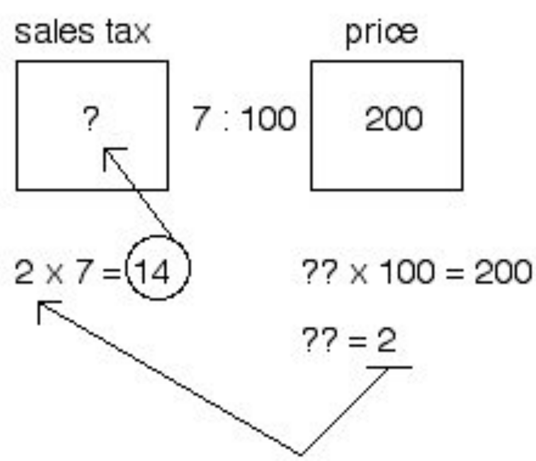
Consider the sand & water problem solved by using a table. 5 was dumped in a certain number of times to reach 30. How many times was 5 dumped in? This is the key to the short cut method. The question of how many times when written as a number sentence is: ?? x 5 = 30. The '??' represents the "dump in" number. For the sand & water example, '??' = 6. This means that if 5 is dumped in 6 times, 2 must also be dumped in 6 times. Thus we need 6 x 2 = 12 cups of water.

Here are two more examples of the short cut method.

A recipe for gooh requires 2 cups of water for every 3 kilograms of dry paste. George has 24 kilograms of paste and wants to use all of it up. How much water does he need to mix with the paste?



The sales tax in Manitoba is 7%. Mary buys a present for her friend that has a selling price of $200. How much sales tax must Mary pay?



Refer to: Meaning of ratio if more help is needed.